On the spontaneous breaking of Lorentz invariance

Orfeu Bertolami [a]

[a]Departamento de Física, Instituto Superior Técnico,
Av. Rovisco Pais, 1049 - 001 Lisboa, Portugal

We show that estimates of the Lorentz symmetry violation extracted from ultra-high energy cosmic rays beyond the GZK cut-off set bounds on the parameters of a Lorentz-violating extension of the Standard Model. Moreover, we argue that correlated measurements of the difference in the arrival time of gamma-ray photons and neutrinos emitted from Active Galactic Nuclei or Gamma-Ray Bursts may provide a signature for a possible violation of the Lorentz symmetry. We find that this time delay is energy independent, but that it has a dependence on the chirality of the particles involved.

1. INTRODUCTION

Lorentz invariance is one of the most fundamental symmetries of physics and is an underlying ingredient of all known physical theories. However, more recently, there has been theoretical evidence that, in the realm of string/M-theory, this symmetry may be violated. This naturally poses the question of verifying this violation experimentally. In this contribution[1], we study the implications of a putative violation of Lorentz invariance in the context of a Lorentz-violating extension of the Standard Model (SM) aiming to analyse its impact on the physics of ultra-high energy cosmic rays and the possibility of an astrophysical test of this violation [1].

Actually, in what concerns the latter issue, it is striking that there is convincing evidence that the observed jets of Active Galactic Nuclei (AGN) are efficient cosmic proton accelerators. Furthermore, the photoproduction of neutral pions by accelerated protons is assumed to be the source of the highest-energy photons through which most of the luminosity of the galaxy is emitted. The decay of charged pions with the ensued production of neutrinos is another distinct signature of the proton induced cascades and estimates of the neutrino flux are believed to be fairly model independent [2,3]. Gamma-Ray Bursts (GRB) have been also suggested as a possible source of high-energy neutrinos [4]. Moreover, a deeper understanding of these sources is expected as large area ($\sim km^2$) high-energy neutrino telescopes are under construction (see e.g. [5]). These telescopes will allow obtaining information that is congenially correlated with gamma-ray flares and bursts emitted by AGN and GRB sources. On the other hand, it has already been pointed out that astrophysical observations of faraway sources of gamma radiation could provide important hints on the nature of gravity-induced effects [6–8] and hence on physics beyond the Standard Model (SM). We argue that delay measurements in the arrival time of correlated sources of gamma radiation and high-energy neutrinos can, when considered in the context of a Lorentz-violating extension of the SM [9], help setting relevant limits on the violation of that fundamental symmetry. As we shall see, we can relate our results with the recently discussed limit on the violation of Lorentz symmetry from the observations of high-energy cosmic rays beyond the GKZ cut-off [10].

The idea of dropping the Lorentz symmetry has been sugested long ago. Indeed, a background cosmological vector field has been considered as

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a way to introduce our velocity with respect to a preferred frame of reference into the physical description [11]. It has also been proposed, based on the behaviour of the renormalization group $\beta-$function of non-abelian gauge theories, that Lorentz invariance could be actually a low-energy symmetry [12]. In higher dimensional theories of gravity, models that are not locally Lorentz invariant have been studied in order to obtain light fermions in chiral representations [13].

The spontaneous breaking of Lorentz symmetry due to non-trivial solutions of string field theory was first discussed in Refs. [14]. These non-trivial solutions arise in the context of the string field theory of open strings and may have striking implications at low-energy. For example, assuming that the contribution of Lorentz-violating interactions to the vacuum energy is about half of the critical density leads to the conclusion that quite feeble tensor mediated interactions in the range of about $10^{-4}\ m$ should exist [15]. Lorentz violation may lie, with the help of inflation, at the origin of the primordial magnetic fields required to explain the observed galactic magnetic field as it may also imply in the breaking of conformal symmetry of electromagnetism [16]. It is quite natural that violations of the Lorentz invariance may imply in the breaking of CPT symmetry [17]. Interestingly, this possibility can be verified experimentally in neutral-meson [19] experiments, Penning-trap measurements [20] and hydrogen-antihydrogen spectroscopy [21]. Moreover, the breaking of CPT symmetry also allows for an explanation of the baryon asymmetry of the Universe, as tensor-fermion-fermion interactions expected in the low-energy limit of string field theories give rise to a chemical potential that creates in equilibrium a baryon-antibaryon asymmetry in the presence of baryon number violating interactions [22].

Limits on the violation of Lorentz symmetry have been directly investigated through laser interferometric versions of the Michelson-Morley experiment where comparison between the velocity of light, c, and the maximum attainable velocity of massive particles, c_i, up to $\delta \equiv |c^2/c_i^2 - 1| < 10^{-9}$ [23]. In the so-called Hughes-Drever experiment [24,25] much more stringent limits can be obtained, searching for a time dependence of the quadrupole splitting of nuclear Zeeman levels along Earth's orbit, e.g. $\delta < 3 \times 10^{-22}$ [26]. Impressively, a more recent assessment of these experiments reveals that more accurate bounds, up to 8 orders of magnitude, can be reached [27]. From the astrophysical side, limits on the violation of momentum conservation and the existence of a preferred reference frame can also be established from bounds on the parametrized post-Newtonian parameter, α_3. This parameter vanishes in General Relativity and can be extracted from the pulse period of pulsars and millisecond pulsars [28]. The most recent bound, $|\alpha_3| < 2.2 \times 10^{-20}$ [29], indicates that Lorentz symmetry is unbroken up to that level.

In the next sections, we shall calculate the corrections to the dispersion relation arising from a Lorentz-violating extension of the SM and confront it with the evidence on the violation of Lorentz invariance arising from the ultra-high energy cosmic ray physics. We shall also see that these corrections induce a time delay in the arrival of signals from faraway sources carried by different particles.

2. LORENTZ-VIOLATING EXTENSION OF THE STANDARD MODEL

It is widely believed that SM is a low-energy description of a more fundamental theory, where all interactions including gravity are unified and the hierarchy problem is solved. It is quite plausible that, in this most likely higher-dimensional fundamental theory, symmetries such as CPT and Lorentz invariance, may undergo spontaneous symmetry breaking. The fact that within string/M-theory, currently the most promising candidate for a fundamental theory, a mechanism where spontaneous breaking of Lorentz symmetry is known [14,17,18], suggests that the violation of those symmetries might actually occur and that its implications should be investigated.

There is no reason, at least in principle, for this breaking not to extend into the four-dimensional spacetime. If this is indeed the case, CPT and Lorentz symmetry violations will be likely to take place within the SM and its effects might be detected. In order to account for the CPT and Lorentz-violating effects an extension to the minimal SM has been developed [9] based on the assumption that CPT and Lorentz-violating terms might arise from interactions of tensor fields to Dirac fields when Lorentz tensors acquire non-vanishing vacuum expectation values. Interactions of this form are expected to arise from the string field trilinear self-interaction, as is in the open string field theory [14,17]. These interactions may also emerge from the scenario where our world is wrapped in a 3-brane and this is allowed to tilt [18]. In order to preserve the SM power-counting renormalizability only terms involving operators of mass dimension four or less are considered in this extention. In [1], only the fermionic sector of the extension [9] was considered [2]. This sector includes both leptons and quarks, since SU(3) symmetry ensures violating extensions to be colour-independent. The extended fermionic sector consists of CPT-odd and CPT-even contributions, which are given by [9]

$$\mathcal{L}^{\mathrm{CPT-odd}}_{\mathrm{Fermion}} = -a_\mu \overline{\psi}\gamma^\mu\psi - b_\mu\overline{\psi}\gamma_5\gamma^\mu\psi \quad , \tag{1}$$

$$\begin{aligned}\mathcal{L}^{\mathrm{CPT-even}}_{\mathrm{Fermion}} &= \tfrac{1}{2} i c_{\mu\nu}\overline{\psi}\gamma^\mu \overleftrightarrow{\partial^\mu}\, \psi \\ &+ \tfrac{1}{2} i d_{\mu\nu}\overline{\psi}\gamma_5\gamma^\mu \overleftrightarrow{\partial^\mu}\, \psi \\ &- H_{\mu\nu}\overline{\psi}\sigma^{\mu\nu}\psi \quad ,\end{aligned} \tag{2}$$

where the coupling coefficients a_μ end b_μ have dimensions of mass, $c_{\mu\nu}$ and $d_{\mu\nu}$ are dimensionless and can have both symmetric and anti-symmetric components, and $H_{\mu\nu}$ has dimension of mass and is anti-symmetric. All Lorentz violating parameters are hermitian and flavour-dependent. Some of them may induce flavour changing neutral currents when non-diagonal in flavour.

[2]It was assumed that SM gauge sector is unaltered. Changing this sector has been already considered, but the phenomenological restrictions are quite severe, at least in what concerns the term that gives origin to a cosmological birefringence [30].

The Langrangian density of the fermionic sector including Lorentz-violating terms reads:

$$\begin{aligned}\mathcal{L} &= \tfrac{1}{2} i\overline{\psi}\,\gamma_\mu \overleftrightarrow{\partial^\mu}\,\psi - a_\mu\overline{\psi}\,\gamma^\mu\psi - b_\mu\overline{\psi}\,\gamma_5\gamma^\mu\psi \\ &+ \tfrac{1}{2} i c_{\mu\nu}\overline{\psi}\,\gamma^\mu \overleftrightarrow{\partial^\nu}\,\psi + \tfrac{1}{2} i d_{\mu\nu}\overline{\psi}\,\gamma_5\gamma^\mu \overleftrightarrow{\partial^\nu}\,\psi \\ &- H_{\mu\nu}\overline{\psi}\,\sigma^{\mu\nu}\psi - m\overline{\psi}\psi \quad ,\end{aligned} \tag{3}$$

where only kinetic terms are kept as we are interested in deducing the free particle energy-momentum relation.

From the above Lagrangian density, we can get the Dirac-type equation

$$\begin{aligned}&[i\gamma^\mu(\partial_\mu + (c_\mu{}^\alpha - d_\mu{}^\alpha\gamma_5)\,\partial_\alpha) - a_\mu\gamma^\mu \\ &-b_\mu\gamma_5\gamma^\mu - H_{\mu\nu}\sigma^{\mu\nu} - m]\psi = 0 \quad .\end{aligned} \tag{4}$$

In order to obtain the corresponding Klein-Gordon equation, we multiply eq. (2) from the left by itself with an opposite mass sign yielding:

$$\begin{aligned}&\Big[[i(\partial_\mu + c^\alpha_\mu\partial_\alpha) - a_\mu]^2 + (d^\alpha_\mu\partial_\alpha)^2 - b^2 - m^2 \\ &- i\sigma^{\mu\rho}[i\partial_\mu c_\rho{}^\beta\partial_\beta + ic_\mu{}^\alpha\partial_\alpha i\partial_\rho + ic_\mu{}^\alpha\partial_\alpha ic_\rho{}^\beta\partial_\beta \\ &- i(\partial_\mu + c_\mu{}^\alpha\partial_\alpha)id_\rho{}^\beta\gamma_5\partial_\beta + id_\mu{}^\alpha\gamma_5\partial_\alpha i(\partial_\rho \\ &+ c_\rho{}^\beta\partial_\beta) - 2b_\mu\gamma_5[i(\partial_\rho + c^\beta_\rho\partial_\beta) - a_\rho]] \\ &- 2i(ia_\mu\sigma^{\mu\rho} - b_\mu\gamma_5 g^{\mu\rho})d_\rho{}^\beta\gamma_5\partial_\beta \\ &+ \sigma^{\mu\nu}\sigma^{\rho\sigma}H_{\mu\nu}H_{\rho\sigma} - H_{\rho\sigma}(\gamma^\mu\sigma^{\rho\sigma} + \sigma^{\rho\sigma}\gamma^\mu) \\ &\quad [i(\partial_\mu + (c_\mu{}^\alpha - d_\mu{}^\alpha\gamma_5)\partial_\alpha) \\ &- a_\mu + b_\mu\gamma_5]\Big]\psi = 0 \quad .\end{aligned} \tag{5}$$

To eliminate the off-diagonal terms, the squaring procedure has to be repeated once again. However, since Lorentz symmetry breaking effects are quite constrained experimentally, violating terms higher than second order will be dropped. After some algebra, we find that off-diagonal terms cannot be fully cancelled, but that these terms

are higher order in the Lorentz violating parameters. To simplify further we also drop $H_{\mu\nu}$. This is justifiable as only time-like Lorentz-violating parameters are going to be studied. Hence, we obtain, for the Klein-Gordon type equation, up to second order in the new parameters:

$$\begin{aligned} & \Big[[(i\partial)^2 + 2i\partial_\mu i c^{\mu\alpha}\partial_\alpha - 2i\partial_\mu a^\mu - m^2]^2 \\ + & \quad 4i\partial_\mu i d_\rho^{\ \beta}\partial_\beta i\partial_\eta i d_\phi^{\ \delta}\partial_\delta (g^{\mu\eta}g^{\rho\phi} - g^{\mu\rho}g^{\eta\phi}) \\ - & \quad 8i\partial_\mu i d_\rho^{\ \beta}\partial_\beta b_\eta i\partial_\phi (g^{\mu\rho}g^{\eta\phi} - g^{\mu\phi}g^{\rho\eta}) \\ + & \quad 4b_\mu b_\eta i\partial_\rho i\partial_\phi (g^{\mu\eta}g^{\rho\phi} - g^{\mu\rho}g^{\eta\phi}) \Big] \psi = 0 \quad . \end{aligned} \tag{6}$$

Thus, in momentum space we get at lowest non-trivial order, the following relationship:

$$\begin{aligned} & (p_\mu p^\mu + 2p_\mu c^{\mu\alpha}p_\alpha + 2p_\mu a^\mu - m^2)^2 \\ + & \quad 4[p_\mu p^\mu d_\rho^{\ \beta} p_\beta d^{\rho\delta} p_\delta - (p_\mu d^{\mu\beta}p_\beta)^2] \\ + & \quad 8(p_\mu p^\mu d_\eta^{\ \beta} p_\beta b^\eta - p_\mu d^{\mu\beta} p_\beta b_\eta p^\eta) \\ + & \quad 4[b_\mu b^\mu p_\nu p^\nu - (b_\mu p^\mu)^2] = 0 \quad . \end{aligned} \tag{7}$$

Hence, the dispersion relation arising from the Lorentz-violating extension of the SM is the following:

$$\begin{aligned} & p_\mu p^\mu - m^2 = -2p_\mu c^{\mu\alpha}p_\alpha - 2p_\mu a^\mu \\ \pm & \quad 2\Big[(p_\mu d^{\mu\beta}p_\beta)^2 - p_\mu p^\mu d_\eta^{\ \beta} p_\beta d^{\eta\delta} p_\delta \\ + & \quad 2(p_\mu d^{\mu\beta}p_\beta b_\rho p^\rho - p_\mu p^\mu d_\eta^{\ \beta} p_\beta b^\eta) \\ - & \quad b_\mu b^\mu p_\nu p^\nu + (b_\mu p^\mu)^2 \Big]^{1/2} \quad , \end{aligned} \tag{8}$$

where the $\pm$ sign refers to the fact that the effects of b_μ and $d_{\mu\nu}$ depend on chirality.

Finally, we consider, for simplicity, the scenario where coefficients a_μ, b_μ, $c_{\mu\nu}$ and $d_{\mu\nu}$ have only time-like components, which yields the simplified dispersion relation

$$p_\mu p^\mu - m^2 = -2c_{00}E^2 - 2aE \pm 2(b + d_{00}E)p \ , \tag{9}$$

where we have dropped the component indices of coefficients a and b. From now on we drop parameter a as it is potentially dangerous from the point of view of giving origin to flavour changing neutral currents when more than one flavour is involved.

In the next section, we shall use the dispersion relation (9) to examine how GZK cut-off for ultra-high energy cosmic rays can be relaxed. The ensued discussion is similar to the one described in [10], where it is assumed that the limiting velocities of particles in different reference frames are *ad hoc* different.

3. ULTRA-HIGH ENERGY COSMIC RAYS

The discovery of the cosmic background radiation made inevitable the question of how the most energetic cosmic-ray particles would be affected by the interaction with the microwave photons. Actually, the propagation of the ultra-high energy nucleons is limited by inelastic impacts with photons of the background radiation so that nucleons with energies above $5 \times 10^{19}\ eV$ are unable to reach Earth from further than $50 - 100\ Mpc$. This is the well known GZK cut-off [31]. However, events where the estimated energy of the cosmic primaries is beyond the GZK cut-off have been observed by different collaborations [34–37]. It has been suggested [10] (see also [33]) that slight violations of Lorentz invariance could be at the origin of energy-dependent effects which would suppress processes, otherwise dynamically inevitable, such as for instance the resonant reaction,

$$p + \gamma_{2.73K} \rightarrow \Delta_{1232} \quad , \tag{10}$$

which is central to the GZK cut-off. Were this process untenable, the GZK cut-off would not hold and therefore a cosmological origin for the high-energy cosmic radiation is theoretically acceptable. As discussed in [10], this can occur through a change in the dispersion relation for free particles. We shall see that this is indeed

what happens when process (10) is analysed with dispersion relation (9). Considering a head-on impact of a proton of energy E with a cosmic background radiation photon of energy ω, the likelihood of the process (10) would be conditioned on satisfying (cf. eq. (9))

$$2\omega + E \geq m_\Delta(1 - c_{00}^\Delta) \quad . \tag{11}$$

Thus, we get from (9) after squaring (11) and dropping the ω^2 term

$$2\omega + \frac{m_p^2}{2E} \geq (c_{00}^p - c_{00}^\Delta)E + \frac{m_\Delta^2}{2E} \quad , \tag{12}$$

which clearly exhibits Lorentz-violating terms.

Let us now compare (12) with the results of Ref. [10] and show that this leads to a bound on Δc_{00}. In order to modify the usual dispersion relation for free particles, Coleman and Glashow suggested assigning a maximal attainable velocity to each particle. Thus, for a given particle i moving freely in the preferred frame, which could be thought of as the one in relation to which the cosmic background radiation is isotropic, the dispersion relation would be

$$E^2 = p^2 c_i^2 + m_i^2 c_i^4 \quad . \tag{13}$$

Hence, the likelihood of the process (10) to occur under the conditions described above would depend on satisfying the kinematical requirement $2\omega + E \geq m_{eff}$, where the effective mass m_{eff} is given by

$$m_{eff}^2 \equiv m_\Delta^2 - (c_p^2 - c_\Delta^2)p^2 \quad , \tag{14}$$

the momentum being with respect to the preferred frame.

The likelihood condition takes then the following form

$$2\omega + \frac{m_p^2}{2E} \geq (c_p - c_\Delta)E + \frac{m_\Delta^2}{2E} \quad , \tag{15}$$

where the term proportional to $c_p - c_\Delta$ is clearly Lorentz-violating. If the difference in the maximal velocities exceeds the critical value

$$\delta(\omega) = \frac{2\omega^2}{m_\Delta^2 - m_p^2} \quad , \tag{16}$$

then reaction (10) would be forbidden and consequently the GZK cut-off relaxed. For photons of the microwave background, $T = 2.73\ K$, and $\omega_0 \equiv kT = 2.35 \times 10^{-4}\ eV$, this condition would be

$$c_p - c_\Delta = \delta(\omega_0) \simeq 1.7 \times 10^{-25} \quad , \tag{17}$$

which is a quite impressive bound on the violation of the Lorentz symmetry, even though valid only for the process in question. Similar bounds for other particle pairs, although less stringent, were discussed in [10,32].

Finally, comparison of (15) with (12) yields:

$$c_{00}^p - c_{00}^\Delta \simeq 1.7 \times 10^{-25} \quad . \tag{18}$$

Thus, we see that the Lorentz-violating extension of the SM can also describe the phenomenology of ultra-high energy cosmic rays and explain the violation of the GZK cut-off. Of course, the situation would be more complex if the Lorentz-violating parameters were allowed to have space-like components which would lead to direction and helicity-dependent effects.

4. AN ASTROPHYSICAL TEST OF LORENTZ INVARIANCE

We now turn to the discussion of a possible astrophysical test of Lorentz invariance. From eq. (18) we see that $\Delta c_{00} \simeq \epsilon$, where ϵ is a small constant specific of the process involved (cf. eqs. (17) and (18)) and, for instance, $|\epsilon| \lesssim few \times 10^{-22}$ from the search of neutrinos oscillations [38,39]. As far as signals simultaneously emitted by far-away sources are concerned, the resulting effect

in the propagation velocity of particles with energy, E, and momentum, p, is given in the limit where $m << p, E$ or, for massless particles, by $c_i = c[1 - (c_{00} \pm d_{00})_i]$. Therefore, for sources at a distance D, the delay in the arrival time will be given by:

$$\Delta t \simeq \frac{D}{c}[(c_{00} \pm d_{00})_i - (c_{00} \pm d_{00})_j] \equiv \epsilon^{\pm}_{ij}\frac{D}{c} \quad , (19)$$

where a new constant, $\epsilon^{\pm}_{ij}$, involving a pair of particles is defined. This time delay may, despite being given by the difference between two fairly small numbers, be measurable for sufficiently far away sources. Furthermore, our result shows that the estimated time delay is energy independent, in opposition to what was obtained from general arguments [6,8]. We have also found that the time delay has a dependence on the chirality of the particles involved. In the next section we shall discuss how to estimate the observational value of c^i_{00} (and d^i_{00} if $d^i_{00} \sim c^i_{00}$).

Therefore if, for instance, the signals from faraway sources were, as suggested previously in the introduction, from AGN TeV gamma-ray flares and the intrinsically related neutrino emission, we should expect for the time delay,

$$\Delta t \simeq (c_{00} \pm d_{00})_\nu \frac{D}{c} \quad , \qquad (20)$$

if as argued above the photon propagation is unaltered and assuming that the neutrinos are massless, an issue that will be most probably settled experimentally in the near future. It is worth stressing that even before that, the effect of neutrino masses and other intrinsic effects related with the nature of the neutrino emission can, at least in principle, be extracted from data of several correlated detections of TeV gamma-ray flares and neutrinos, if a systematic delay of neutrinos is observed. Moreover, the available knowledge of AGN phenomena and the confidence on the astrophysical methods available to determine their distance from us, make it plausible that the time delay strategy may provide relevant bounds on the violation of Lorentz symmetry. Of course, similar arguments may equally apply to GRB, however, the lack of a deeper understanding of these transient phenomena introduces further undesirable uncertainty. It is also important to point out that bounds involving photon and neutrinos are currently unknown and that a difference in the arrival time between neutrinos and antineutrinos is expected if d^i_{00} is non-vanishing.

5. CONCLUSIONS

In summary, we have shown that parameters of the Lorentz-violating extension of the SM proposed in Ref. [9] can be related with the phenomenology of ultra-high energy cosmic rays with the conclusion that, as in [10], it may lead to the suppression of processes responsible for the GZK cut-off. This is a crucial argument in favour of a possible extra-galactic origin for the ultra-high energy cosmic rays. We have also found that the relevant Lorentz-violating parameter is, at high energies, c^i_{00} so that $\Delta c_{00} \simeq \epsilon$ with $|\epsilon| \lesssim few \times 10^{-22}$ from neutrino physics and $|\epsilon| \lesssim 10^{-25}$ from the ultra-high energy cosmic ray physics. It is possible to estimate the typical scales assuming that the source of Lorentz symmetry violation is due to non-trivial solutions in string field theory. Indeed, these solutions imply that Lorentz tensors acquire vacuum expection values as Lorentz symmetry is spontaneously broken due to string induced interactions [14,17]. A parametrization for these expectation values and, hence, for c^i_{00} would be for a fixed energy scale, E, the following [17,22]:

$$c^i_{00} \simeq \frac{\langle T \rangle}{M_S} = \lambda_i \left(\frac{m_L}{M_S}\right)^l \left(\frac{E}{M_S}\right)^k \quad , \qquad (21)$$

where T denotes a generic Lorentz tensor, λ_i is presumably an order one flavour-dependent constant[3], m_L is a light mass scale, M_S is a string scale presumably close to Planck's mass or a few orders of magnitude below it, and k, l are integers labelling the order of the string corrections

[3]A scenario were λ_i is of order of the respective Yukawa coupling has been also discussed [17].

at low-energy. Thus, in the lowest non-trivial order, $k = 0,\ l = 1$ ($k = l = 0$ being already excluded experimentally), $c^i_{00} = \lambda_i \left(\frac{m_L}{M_S}\right)$ and different λ_i constants lead to $\epsilon \simeq \left(\frac{m_L}{M_S}\right)$. Thus, we see that theoretical estimates are consistent with high-energy cosmic ray data. Moreover, if for example, $\epsilon \lesssim 10^{-23}$, then it follows that $m_L \sim 10^2\ KeV$ for $M_S \simeq M_P$ or $m_L \sim 10^2\ eV$ if $M_S \simeq few \times 10^{16}\ GeV$ [40]. Estimates for m_L would clearly change by many orders of magnitude if λ_i were of order of the Yukawa coupling. In either case, we can conclude that choice $k = 0,\ l = 1$ implies the time delay in the arrival of signals from faraway sources is energy independent. We have found, however, an interesting dependence on the chirality of particles involved.

Of course, another scenario would emerge from a different choice of integers k, l. For instance, the choice $k = 2$ and $l = 0$, the relevant one in the CPT symmetry violating baryogenesis scenario [22], where the energy should, in this case, be related with the early Universe temperature. This would imply that the time delay in the arrival of signals from faraway sources would be proportional to the square of the energy. This choice would also lead to the conclusion that Lorentz violating effects, whether due to string physics or quantum gravity, are quadratic in the energy. Similar conclusions concerning the order of quantum gravity low-energy effects are obtained from the study of corrections to the Schrödinger equation arising from quantum cosmology in the minisuperspace approximation [41].

In another theoretical setting, the spontaneous breaking of the Lorentz symmetry may occur is the so-called braneworld [18]. In this scenario, SM particles lie on a 3-brane, $\phi(x)$, embedded in spacetime, with possibly large compact extra dimensions, whereas gravity propagates in the bulk. Thus, a tilted brane would induce rotational and Lorentz non-invariant terms in the four-dimensional effective theory as brane-Goldstones couple to all particles on the brane via an induced metric on the brane. This will lead to operators of the form:

$$\partial_\mu \phi\ \partial_\nu \phi\ \overline{\psi}\ \gamma^\mu \partial^\nu \psi + ... \quad , \tag{22}$$

which clearly resemble some of the Lorentz-violating terms in the SM extention discussed above. Corrections to the kinetic terms of gauge fields are also expected. As before, phenomenology sets tight constraints on this scenario remaining, however, unable to establish whether the breaking of Lorentz invariance, if observed at all, has its origin on the non-perturbative nature of branes or has its roots in the perturbative string field theory scenario described above. The former possibility should more probably be associated with a M_S scale that is a few orders of magnitude below Planck scale, while the latter to a value of M_S that should be associated with the Planck scale itself.

Finally, we have outlined a strategy to establish to what extent Lorentz invariance is violated, from the observation of the time delay in the detection of TeV gamma-ray flares and neutrinos from AGN. Our analysis reveals that the time delay has a dependence on the chirality of the particles involved, but is energy independent, contrary to what one could expect from general arguments. In either case, if ever observed, a time delay in the arrival of signals from faraway sources would be a strong evidence of new physics beyond the SM.

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